Homework set # 4

Due on 2/13

0. The following problems from Artin “Algebra” edition 2: 15.4.2; 15.7.5; 15.7.8

1. Determine the splitting field and its degree over $\mathbb{Q}$ for $x^4 - 2$.

2. For any prime p and any nonzero $a \in \mathbb{F}_p$ prove that $x^p - x + a$ is irreducible and separable over $\mathbb{F}_p$. Recall that a polynomial is separable over a field if it has no multiple roots over the given field. (Hint: For irreducibility you can either first prove that if α is a root then $\alpha + 1$ is also a root. Or you could suppose that it is irreducible and compute derivatives.)

3. Prove that

$$x^{p^n-1} - 1 = \prod_{\alpha \in \mathbb{F}_{p^n}^\times} (x - \alpha).$$

Conclude that (i.e. give an argument for why)

$$\prod_{\alpha \in \mathbb{F}_{p^n}^\times} \alpha = (-1)^{p^n}$$

so the product of nonzero elements of a finite field is 1 if $p = 2$ and -1 if p is odd.