# Homework set # 2

**Due on 1/30**

**0.** The following problems from Artin "Algebra" edition 2: 12.4.3; 15.2.1; 15.3.1; 15.3.2

**1.** Prove that a polynomial of degree 2 or 3 over a field $F$ (i.e. in $F[x]$) is reducible if and only if it has a root in $F$. (Hint: First show that $p(x) \in F[x]$ has a factor of degree 1 in $F[x]$ if and only if $p(x)$ has a root in $F$.)

**2.** Show that the polynomial $(x-1)(x-2)\cdots(x-n)-1$ is irreducible over $\mathbb{Z}$ for all integers $n \geq 1$. (Hint: consider the values of the polynomial at $1, 2, \ldots, n$)

**3.** Let $K$ be an extension field of $F$. Suppose the degree of $K$ over $F$ is a prime $p$. Show that any subfield $E$ of $K$ containing $F$ is either $K$ or $F$.

**4.** Prove that if $[F(\alpha) : F]$ is odd then $F(\alpha) = F(\alpha^2)$.