Hyperfine Quenching: Review of Experiment and Theory

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I Two approaches: Perturbation Theory & Radiation Damping

II Heliumlike Ions

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IV Ni-like Ions and Other Systems

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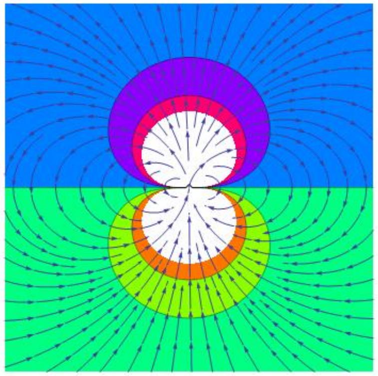
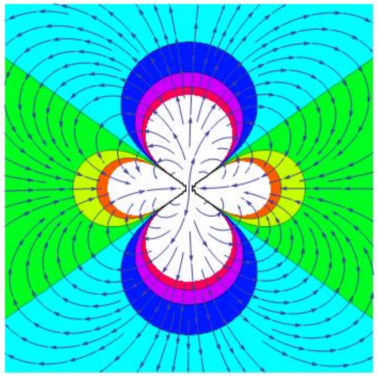
Hyperfine Interaction





$$H_1 = \frac{e}{c}\boldsymbol{\alpha} \cdot \left[\frac{\boldsymbol{\mu}_I \times \boldsymbol{r}}{r^3}\right] \qquad H_2 = -e\left[\frac{Q_{ij} r_i r_j}{6\, r^3}\right]$$

$$\begin{aligned} H_{\rm hfs} &= e\left[\sum_{\lambda=1}^{3} (-1)^\lambda \frac{i\sqrt{2}\,[\boldsymbol{\alpha}\cdot\vec{C}^{(0)}_{1\lambda}(\hat{r})]}{cr^2}\,\mu_{-\lambda} + \sum_{\lambda=1}^{5}(-1)^\lambda \frac{C^2_\lambda(\hat{r})}{r^3} Q_{-\lambda}\right] \\ &= \sum_k T_k^{(e)} \cdot T_k^{(n)} \end{aligned}$$

Perturbation Theory

$$H_{\rm hfs} = \sum_k T_k^{(e)} \cdot T_k^{(n)}$$

1) $\boldsymbol{F} = \boldsymbol{I} + \boldsymbol{J}$ is conserved.
2) Level J splits into sublevels with $F{=}I + J$, $F{=}I + J - 1$, $\cdots$ $F{=}|I - J|$.
3) Assuming fine structure levels well spaced

$$W_J^F = E_J + (-1)^{I+J+F} \sum_k \left\{ \begin{array}{ccc} I & J & F \\ J & I & k \end{array} \right\} \langle J||T_k^{(e)}||J\rangle \langle I||T_k^{(n)}||I\rangle$$

4) Assuming several closely spaced levels γJ, we have

$$W_{\gamma J,\gamma' J'}^F = E_{\gamma J}\, \delta_{\gamma\gamma'}\, \delta_{JJ'} + (-1)^{I+J+F} \sum_{k\, \gamma' J'} \left\{ \begin{array}{ccc} I & J & F \\ J' & I & k \end{array} \right\} \langle \gamma J||T_k^{(e)}||\gamma' J'\rangle \langle I||T_k^{(n)}||I\rangle$$

Perturbation Theory (Continued)

5) Wave Function:

$$\Psi_{\gamma J} \;\longrightarrow \Psi_{\gamma J} + \delta\Psi^F_{\gamma J}$$

$$\delta\Psi^F_{\gamma J} = \;(-1)^{I+J+F}\sum_{k\,\gamma' J'}\left\{\begin{array}{ccc} I & J & F \\ J' & I & k \end{array}\right\}\frac{\langle\gamma J||T^{(e)}_k||\gamma' J'\rangle\langle I||T^{(n)}_k||I\rangle}{E_{\gamma J} - E_{\gamma' J'}}\Psi_{\gamma' J'}$$

$$\langle\Psi_{\gamma J}|Q_l|\Psi_0\rangle \longrightarrow \langle\Psi_{\gamma J}|Q_l|\Psi_0\rangle + \langle\delta\Psi^F_{\gamma J}|Q_l|\Psi_0\rangle$$

$$\langle\delta\Psi^F_{\gamma J}|Q_l|\Psi_0\rangle = \sum_{\gamma' J'} C^F_{\gamma' J',\gamma J}\langle\Psi_{\gamma' J'}|Q_l|\Psi_0\rangle$$

Energy Matrix for F = 1/2 in He-like ^{31}P (I=1/2)

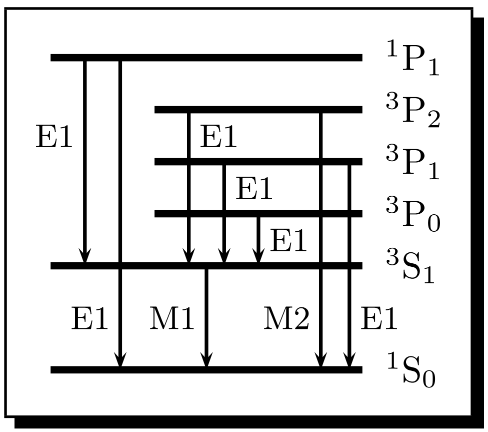
$$W^{F=1/2}_{\gamma J,\gamma' J'} \qquad (1/\text{cm})$$

	$2\,^3P_0$	$2\,^3P_1$	$2\,^3P_2$	$2\,^1P_1$
$2\,^3P_0$	0.0000[0]	2.3384[1]	0.0000[0]	-1.4234[1]
$2\,^3P_1$	2.3384[1]	2.4114[3]	0.0000[0]	2.2012[1]
$2\,^3P_2$	0.0000[0]	0.0000[0]	1.2196[4]	0.0000[0]
$2\,^1P_1$	-1.4234[1]	2.2012[1]	0.0000[0]	1.0183[5]
$W^{1/2}$	-2.2880[-1]	2.4116[3]	1.2196[4]	1.0183[5]
	Eigenvector Matrix			
$2\,^3P_0$	9.9995[-1]	9.6974[-3]	0.0000[0]	-1.3973[-4]
$2\,^3P_1$	-9.6974[-3]	9.9995[-1]	0.0000[0]	2.2138[-4]
$2\,^3P_2$	0.0000[0]	0.0000[0]	1.0000[0]	0.0000[0]
$2\,^1P_1$	1.4187[-4]	-2.2002[-4]	0.0000[0]	1.0000[0]

Transition rates in He-like ^{31}P (I=1/2)

Induced $E1$ transitions from $2\,^3P_{0,2}$ states to ground state

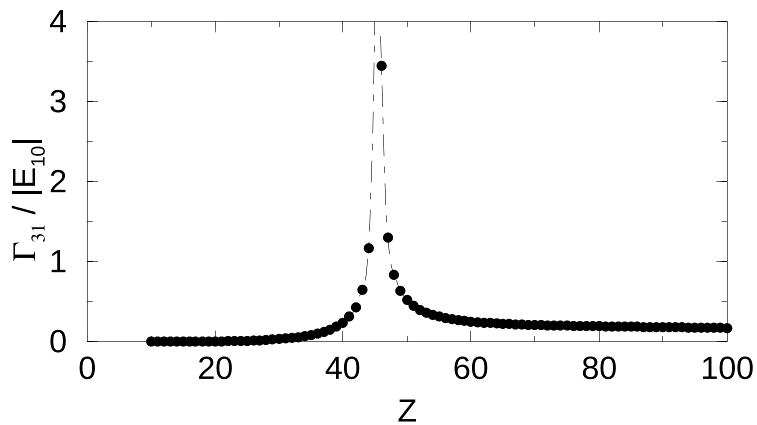
$$A^F_{E1}[^3P_J] = \frac{2.02613 \times 10^{18}}{3(2F+1)\lambda^3} \left| \sum_{\gamma'=1,3} C^F_{\gamma' 1,3J} \langle 1\,^1S_0 \| Q_1 \| 2\,^{\gamma'}P_1 \rangle \right|^2$$



Mode	P_2	P_0
$E1 \longrightarrow 2\,^3S_1$	0.0037	0.0409
$M2 \longrightarrow 1\,^1S_0$	0.0689	0.0000
Induced $E1 \longrightarrow 1\,^1S_0$	0.2214	0.1659
$A_{\rm tot}$	0.2940	0.2068
Expt.	0.298(4)	0.205(4)

Difficulties with Previous Method

A case where above theory does not work: $^3\mathrm{P}_0$ level in heliumlike ^{207}Ag.
Theory: 384.8 (ns^{-1})
Expt: 251±23 (ns^{-1})



Solution: Treat interaction with radiation field on same level as hyperfine interaction $H_{\mathrm{hf}} \rightarrow H_{\mathrm{hf}} + V_{\mathrm{rad}}$

Radiation Damping[1]

$$V_{\rm rad}|\psi_E\rangle = ie^2 \sum_{lq\lambda} \frac{(l+1)(2l+1)}{l\,[(2l+1)!!]^2} \sum_n k_n^{2l+1} Q_{lq}^{(\lambda)} |\psi_n\rangle\langle\psi_n| Q_{lq}^{(\lambda)\dagger} |\psi_E\rangle$$

$$\langle 2\,{}^\gamma P_1 | V_{\rm rad} | 2\,{}^\gamma P_1\rangle \approx i\frac{\hbar}{2}\left[A(2\,{}^\gamma P_1 \rightarrow {}^1S_0) + A(2\,{}^\gamma P_1 \rightarrow {}^3S_1)\right]$$

$$\langle 2\,{}^3P_1 | V_{\rm rad} | 2\,{}^1P_1\rangle \approx i\frac{\hbar}{2}\left[\frac{4}{9}\,k_0^3\,\langle 1\,{}^1S_0 \| Q_1 \| 2\,{}^1P_1\rangle\langle 1\,{}^1S_0 \| Q_1 \| 2\,{}^3P_1\rangle \right.$$

$$\left. + \frac{4}{9}\,k_1^3\,\langle 2\,{}^3S_1 \| Q_1 \| 2\,{}^1P_1\rangle\langle 2\,{}^3S_1 \| Q_1 \| 2\,{}^3P_1\rangle\right]$$

[1] F. Robicheaux et al., Phys. Rev. A **52**. 1319 (1995).

Radiation Damping for He-lke ^{107}Ag

Eigenvalues of $(H_0 + H_{\mathsf{hf}} + iV_{\mathsf{rad}})_{\gamma J, \gamma' J'}$ for $F = 1/2$.

$^3P_0$	$^3P_1$	$^3P_2$	$^3P_1$
$\Re$ Eigenvalues in cm^{-1}			
1.0699[0]	-6.3188[3]	1.6955[6]	1.9426[6]
$2\Im$ Eigenvalues in ns^{-1}			
2.6851[2]	1.5807[6]	7.8768[2]	3.8868[6]

$\Psi(2\ ^3P_0)(t) \sim \exp[i(W^{(1/2)} + i\Gamma^{(1/2)}/2)t]$

$\Gamma^{(1/2)} = 268.5$ ns^{-1}

$\Gamma_{\rm Expt} = 251 \pm 23$ ns^{-1}

Hyperfine Quenching for He-like Ions (Theory)

- PT 1/Z calculations of quenching rates of $^3P_{0,2}$ levels for He-like ions.[2]
- RD MCDF calculations of quenching rates of $^3P_0$ levels for He-like ions with Z=46-92 including the Breit interaction and the Lamb shift.[3]
- PT MCHF calculations of quenching rates of $^3P_0$ levels for He-like F, Na and Al including Breit-Pauli corrections.[4]
- RD Relativistic CI calculations of $^3P_{0,2}$ levels for He-like ions with Z=6-100 including the Breit interaction and Lamb shift.[5]
- PT 1/Z calculations of quenching rates of $^3P_0$ levels for He-like ions including Coulomb and Breit interactions.[6]

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[2] P. J. Mohr in *Beam-Foil Spectroscopy*, Vol.1, pp. 9-103 (1976).

[3] P. Indelicato et al., Phys. Rev. A **40**, 3505 (1989).

[4] A. Aboussaïd et al. Phys. Rev. A **51**, 2031 (1995).

[5] W. Johnson, K.T. Cheng and D. Plante, Phys. Rev. A **55**, 2728 (1997).

[6] A. Volotka et al. Can. J. of Phys. **80**, 1263 (2002).

$(1s2p)\ ^3P_0$ Decay in He-like Ions

Comparison of theory & experiment for lifetimes (ns)) of $(1s2p)\ ^3P_0$ in heliumlike ions.

Ion	I	μ_I	Expt.	Theory	Ref.
^{19}F	1/2	2.6289	9.48(20)	9.574	Engsrtröm et al.
^{27}Al	5/2	3.6415	4.80(20)	4.695	Denne et al.
^{31}P	1/2	1.1316	4.88(9)	4.836	Livingston & Hinterlong
^{61}Ni	3/2	-0.75002	0.47(5)	0.4455	Dunford et al.
^{107}Ag	1/2	-0.11368	0.00398(37)	0.003724	Marrus et al.
^{109}Ag	1/2	-0.13069	0.00284(32)	0.002810	Simionovici et al.
^{155}Gd	3/2	-0.25810	0.01343(27)	0.01357	Indelicato et al.
^{157}Gd	3/2	-0.33860	0.00765(55)	0.00801	Indelicato et al.
^{197}Au	3/2	0.14816	0.02216(81)	0.002261	Toleikis et al.

Determination of Fine-Structure Interval

For high Z, the decay rate of the $(1s2p)^3\mathrm{P}_0$ state is determined (approx.) by the eigenvalues of the matrix[7]

$$\begin{pmatrix} \langle ^3P_0|H_{\mathsf{hf}} + iV_{\mathsf{rad}}|^3P_0\rangle & \langle ^3P_0|H_{\mathsf{hf}} + iV_{\mathsf{rad}}|^3P_1\rangle \\ \langle ^3P_1|H_{\mathsf{hf}} + iV_{\mathsf{rad}}|^3P_0\rangle & E_{10} + \langle ^3P_1|H_{\mathsf{hf}} + iV_{\mathsf{rad}}|^3P_1\rangle \end{pmatrix}$$

where E_{10} is the interval between the $^3P_1$ and $^3P_0$ states. E_{10} is treated as an adjustable parameter to give the observed rate.

	Expt.	Theor.	Ref.
Ag	0.79(04)	0.801	Birkett et al.
Gd	18.57(19)	18.57	Indelicato et al.

[7]Indelicato,et al., Phys. Rev. A **40**, 3505, (1989).

$(1s2p)\ ^3P_2$ Decay in He-like Ions

$^3P_2$ rates (ns^{-1}): Cases where HF contribution $> 5\%$ and $\tau > 1$ ps.

	μ_I	I	$A_{\text{M2+E1}}$	A_{hf}	A_{tot}	% from HFS
^{45}Sc	4.7565	7/2	1.693	0.3928	2.085	23
^{50}V	3.3457	6	3.188	0.3622	3.550	11
^{51}V	5.1487	7/2	3.188	0.9453	4.133	29
^{51}Mn	3.5683	5/2	5.891	0.9584	6.850	16
^{55}Mn	3.4687	5/2	5.891	0.9056	6.797	15
^{59}Co	4.6270	7/2	10.59	2.733	13.33	25
^{63}Cu	2.2273	3/2	18.49	1.453	19.94	7
^{65}Cu	2.3816	3/2	18.49	1.662	20.15	8
^{69}Ga	2.0166	3/2	31.31	2.035	33.35	6
^{71}Ga	2.5623	3/2	31.31	3.285	34.60	10
^{79}Br	2.1064	3/2	82.74	5.908	88.65	7
^{81}Br	2.2706	3/2	82.74	6.865	89.61	8
^{87}Rb	2.7515	3/2	129.6	15.83	145.4	12
^{93}Nb	6.1705	9/2	298.5	134.9	433.4	45
^{99}Tc	5.6847	9/2	440.6	169.3	609.9	38

$(1s2s)\ ^1S_0$ Decay in He-like Ions

$(1s2p)\ ^3P_0$ and $(1s2s)\ ^1S_0$ levels in He-like ions cross near Z=62 making the ions He-like Eu and He-like Gd interesting for atomic PNC experiments. Quenching of the $(1s2s)\ ^1S_0$ decay caused by mixing with the $(1s2s)\ ^3S_1$ state which decays to the $(1s)^2\ ^1S_0$ via an M1 transition has been evaluated.[8]

Induced M1 transition rates (s^{-1}) of the $(1s2s)\ ^1S_0$ state of He-like Eu and Gd.

	μ_I	I	$A_{\text{M1 hf}}$
^{151}Eu	3.4717	5/2	0.68[8]
^{153}Eu	1.5330	5/2	0.13[8]
^{155}Gd	-0.2591	3/2	0.58[6]
^{157}Gd	-0.3398	3/2	0.99[6]

Effects of magnetic fields and nuclear polarization on this transition have also been studied.[9]

[8] L. N. Labzowsky et al., Phys. Rev. A**63**, 054105,(2001), Li et al., Eur. Phys. J. D **51**, 313 (2009).
[9] Bondarevskaya et al., Phys. Lett. A **322**, 6642, (2008).

$^3P_0 - ^1S_0$ Transitions in Alkaline Earth Atoms

$^3P_0 - ^1S_0$ & $^3P_2 - ^1S_0$ transitions in neutral alkaline earth atoms are candidates for ultra-precise atomic clocks: $A[^3P_0]/\Delta E \sim 10^{-18}$

Quenched rates for $^3P_0$ states in Alkaline Earth Atoms[10]

	I	μ_I	Q (b)	$A[^3P_0]$ (s^{-1})
^{25}Mg	5/2	-0.85546	0.1994(20)	4.44[-4]
^{43}Ca	7/2	-1.31727	-0.0408(8)	2.22[-3]
^{87}Sr	9/2	-1.09283	0.335(20)	7.58[-3]
^{171}Yb	1/2	0.4919		4.35[-2]
^{173}Yb	5/2	-0.6776	2.800(40)	3.85[-2]

A CI + MBPT effective Hamiltonian H = H$_{HF}$ + Σ was used in the calculation.

Comparison: For Be-like Mg, Garstang[11] obtained $A[^3P_0]$ = 4.2[-4] (s^{-1}), However, for $^3P_2$ decay, serious differences arise in the F-dependent results.

[10] S. Porsev and A. Derevianko, Phys. Rev. A**69**, 042506 (2004).

[11] R. H. Garstang, J. Opt. Soc. Am. **52**, 845 (1962)

$^3P_0 - ^1S_0$ Transition Rates (s^{-1}) for Be-like Ions

	Marques[12]	Brage[13]	Cheng[14]	Andersson[15]	Expt.[16,17]
^{15}N	9.47[-5]	3.62[-4]	3.27[-4]	3.27[-4]	4.0(1.3)[-4]
^{47}Ti	0.356		0.673	0.677	0.56(3)

RD MCDF calculations including relativistic and QED corrections but restricted to interaction between $^3P_0$ and $^3P_1$ states.[12]

PT MCDF calculations for Be-like and Mg-like ions.[13]

PT Relativistic CI calculations including Breit and QED corrections of the $^3P_0$ quenching rate for Be-like ions.[14]

PT MCDF calculations of quenching rates for $^3P_0$ and $^3P_2$ states including Breit and QED corrections.[15]

Exp. The experimental value of the $^3P_0$ quenching rate in ^{15}N is determined from observations of planetary nebula NGC3918.[16]

Exp. Resonant electron-ion recombination in a heavy-ion storage ring is employed to monitor the time dependent population of the $^3P_0$ state.[17]

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[12] J. Marques et al., Phys. Rev. A,**47**, 929 (1993).
[13] T. Brage et al., Ap. J. 500, 507 (1998).
[14] K. T. Cheng et al., A **77**, 052504 (2008).
[15] M. Andersson et al.,A **79**, 032501 (2009).
[16] T. Brage et al., Phys. Rev. Lett. **89**, 28(2002).
[17] S. Schippers et al., Phys. Rev. Lett. **98**, 033001 (2007).

${}^3P_0 - {}^1S_0$ Transition Rates (s^{-1}) for Mg-like Ions

	Marques[18]	Kang[20]	Andersson[21]	Expt.[22]
$^{31}Al^+$	2.65[-2]	4.33[-2]	4.40[-2]	4.85(0.2)[-2]

- RD MCDF calculations including Breit and QED corrections using a complex matrix method of quenching rates for $(3s3p)\,{}^3P_0$ levels in Mg-like ions for atoms with nuclear charge Z=14-92.[18]
- PT MCHF calculations with Breit-Pauli corrections of quenching rates for ${}^3P_0$ levels in the Mg-like ions.[19]
- PT MCDF calculations of quenching rates for ${}^3P_0$ states of Mg-like ions (Z=13-78) including Breit and QED corrections. [20]
- PT MCDF calculations including Breit and QED corrections of quenching rates for ${}^3P_0$ and ${}^3P_2$ states of Mg-like ions (Z=12-31).[21]
- Exp. Laser spectroscopy measurement of the lifetime of the ${}^3P_0$ state of Mg-like Al.[22]

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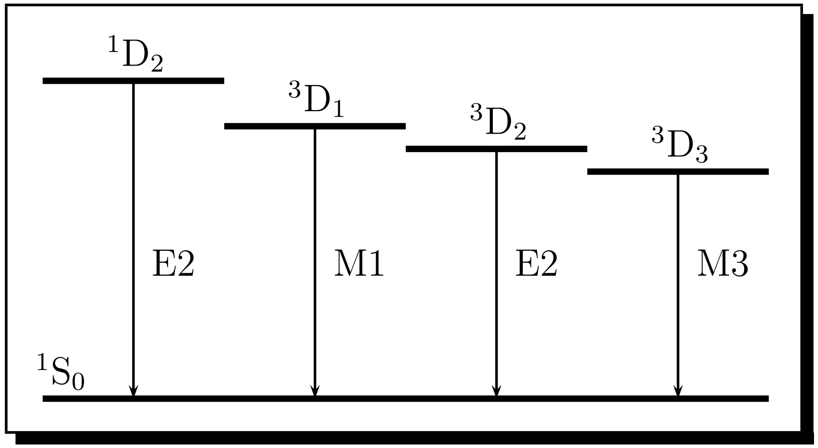
[18] J. Marques et al., At. Data & Nucl. Data Tables **55**, 157 (1993).
[19] T. Brage et al., Ap. J. 500, 507 (1998).
[20] H. Kang et al., J. Phys. B **42**, 195002 (2009).
[21] M. Andersson et al., J. Phys. B **43**, 095001 (2009)
[22] Rosenband et al., Phys. Rev. Lett. **98**, 220801 (2007).

$(3d)^9 4s\ D$ Transitions in Ni-like Ions



For Ni-like ^{129}Xe (I=1/2), only the F=5/2 sublevel of the $^3D_3$ level is quenched by mixing with $^{1,3}D_2$ states; the F=7/2 state decays by M3 emission only.

$$N[^3D_3](t) = N_{7/2}(0)e^{-\Gamma_{M3}\, t} + N_{5/2}(0)e^{-(\Gamma_{M3}+\Gamma_{E2\mathrm{hf}})\, t}$$

Rates determined from the measured decay curve[23] agree very well with MCDF calculations.[24]

[23] Träbert et al., Phys. Rev. Lett. **98**, 263001 (2007); Träbert et al., Phys. Rev. A**73**, 022508 (2006);
[24] Yao et al., Phys. Rev. Lett. **98**, 269304 (2007).

Other Cases

Ti-like Theory Ions in the Ti sequence have a ground state configuration $(4d)^4\ {}^5D_J$. The decay rate for the $J = 4 \rightarrow J' = 0$ transition within the ground multiplet is strongly modified by hyperfine quenching. The rate, which is very sensitive to the nuclear electric-quadrupole moment, can lead to a new method of measuring quadrupole moments.[25]

Zn-like Theory: Hyperfine quenching of $4p4s\ {}^3P_0$ levels in Zn-like ions Z=30-92 using MCDF wave functions and the radiation-damping method.[26] Expt.: Differences between dielectric recombination rate coefficients of even and odd A isotopes of Zn-like Pt were observed and attributed to hyperfine quenching.[27]

Ne-like Theory Calculations of quenching of the $(2p)^5 3s\ {}^3P_2$ and $(2p)^5 3s\ {}^3P_0$ levels of Ne-like ions were made for Z=13-79.[28]

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[25] P. Indelicato, Phys. Scr. T**65**, 57 (1996). & F. Parente et al., Europhys. Lett.**26**, 437 (1994).
[26] J. P. Marques et al., Eur. Phys. J. D**41**, 457 (2007).
[27] Schippers et al., Nucl. Inst. & Methods B**235**, 265 (2005).
[28] M. Andersson et al., J. Phys. Conf. Series **163**, 12013 (2009).

Other Cases (continued)

Ra Theory MCDF calculations of decay of the $7s7p\,^3P_0$ state through 2-photon E1M1 and hyperfine induced channels were made.[29]

XeII Expt. Using a state selective laser probing technique for lifetime measurement in an ion storage ring, evidence for a drastic differences between the decay rates of the hyperfine states of the metastable level $(5p)^4 5d\,^4D_{7/2}$ in ^{129}Xe$^+$ was found.[30]

[29] J. Bieron et al., Eur. Phy. J - Special Topics **144**, 75 (2007),

[30] S. Mannervik et al., Phys. Rev. Lett. 76, 3675 (1996).

Summary and Conclusion

- Two Methods: "perturbation theory" and "radiation damping". The methods agree far away from level crossing. Radiation damping theory is appropriate near level crossing.
- He-like ions thoroughly studied theoretically. Experimental studies of F-dependent decays of $(1s2p)\,^3P_2$ levels would still be of interest.
- Theory is relatively complete for Be-like and Mg-like ions but there are few experiments.
- Experimental and theoretical studies of F-dependent HFQ for Ni-like ions agree well for Ni-like Xe. Similar studies for other ions would build confidence our understanding.
- Interesting possibilities for measuring nuclear quadrupole moments in Ti-like ions.