Parity Nonconservation in Atoms: The Weak Charge and Anapole Moment of ^{133}Cs

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1) Weak charge Q_W of ^{133}Cs provides a test of the Standard Electroweak Model.

2) First (only) observation of an anapole moment κ_a was in ^{133}Cs.

3) Q_W^{exp} and κ_a^{exp} **require** accurate calculations together with **error estimates**!

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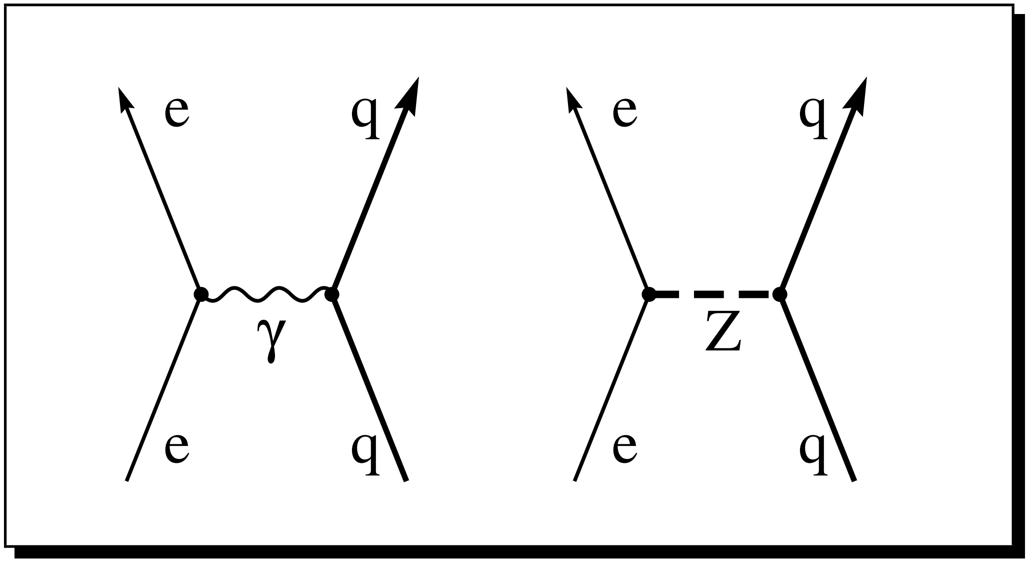
Atomic Parity Nonconservation



A consequence of Z exchange is violation of Laporte's rule:
"Radiative (E_1) transitions take place only between states of opposite parity."

Laporte: http://www.nap.edu/books/0309025494/html/268.html

Otto Laporte (1902-1971) discovered the law of parity conservation in physics. He divided states of the iron spectrum into two classes, even and odd, and found that no radiative transitions occurred between like states.[1]

[1] O. Laporte, Z. Physik **23** 135 (1924).

Z Exchange in the Standard Model[2]

$$H_{\mathsf{PV}} = \frac{G}{\sqrt{2}} \Big[\bar{e}\gamma_\mu\gamma_5 e \left(c_{1u}\, \bar{u}\gamma_\mu u + c_{1d}\, \bar{d}\gamma_\mu d + \cdots \right)$$

$$+ \bar{e}\gamma_\mu e \left(c_{2u}\, \bar{u}\gamma_\mu\gamma_5 u + c_{2d}\, \bar{d}\gamma_\mu\gamma_5 d + \cdots \right) \Big]$$

where $\cdots = t,\, b,\, s,\, c$

$$c_{1u} = -\frac{1}{2} + \frac{4}{3}\ \sin^2\theta_W \qquad c_{1d} = \frac{1}{2} - \frac{2}{3}\ \sin^2\theta_W$$

$$c_{2u} = -\frac{1}{2}\left(1 - 4\ \sin^2\theta_W\right) \qquad c_{2d} = \frac{1}{2}\left(1 - 4\ \sin^2\theta_W\right)$$

[2] W. J. Marciano in *Precision Tests of the Standard Electroweak Model*, Ed. P. Langacker, (World Scientific, Singapore, 1995), p. 170.

Electron Axial-Vector – Nucleon Vector

Contribution of *coherent* vector nucleon current:

$$H^{(1)} = \frac{G}{2\sqrt{2}}\,\gamma_5\,Q_W\,\rho(r)$$

where $\rho(r)$ is a nuclear density ($\sim$ neutron density) and

$$\begin{aligned} Q_W &= 2[(2Z+N)c_{1u} + (Z+2N)c_{1d}] \\ &= -N + Z\,(1 - 4\,\sin^2\theta_W) \\ &\sim -N \end{aligned}$$

Electron Vector – Nucleon Axial-Vector

Contribution of vector axial-vector nucleon current:

$$H^{(2)} = -\frac{G}{\sqrt{2}}\boldsymbol{\alpha}\cdot\left[c_{2p}\left\langle\phi_p^\dagger\boldsymbol{\sigma}\phi_p\right\rangle + c_{2n}\left\langle\phi_n^\dagger\boldsymbol{\sigma}\phi_n\right\rangle\right]$$

where $\langle\cdots\rangle$ designates nuclear matrix elements.

$$c_{2p} \sim 1.25 \times c_{2u} = -0.068$$

$$c_{2n} \sim 1.25 \times c_{2d} = 0.068$$

A) Nucleon Axial-Vector Contribution

$$H^{(2)} = \frac{G}{\sqrt{2}} \kappa_2 \, \alpha \cdot \mathbf{I} \, \rho(r)$$

κ_2 from "Extreme" Shell Model and from Nuclear Calculations.[3]

Element	A	State	κ_2 [Sh. Mod.]	κ_2 [3]
K	39	$1d_{3/2}$ (p)	0.0272	
Cs	133	$1g_{7/2}$ (p)	0.0151	0.0140
Ba	135	$2d_{3/2}$ (n)	-0.0272	
Tl	205	$3s_{1/2}$ (p)	-0.136	-0.127
Fr	209	$1h_{9/2}$ (p)	0.0124	

[3] W. C. Haxton, C.-P. Liu, and M. J. Ramsey-Musolf, Phys. Rev. Lett. **86**, 5247 (2001).

B) Nuclear Anapole Moment Contribution

PNC in nucleus $\Rightarrow$ nuclear anapole:

$$H^{(a)} = e\,\boldsymbol{\alpha}\cdot\boldsymbol{A} \rightarrow \frac{G}{\sqrt{2}}\kappa_a\,\alpha\cdot\mathbf{I}\,\rho(r)$$

Theoretical estimates[4] for ^{133}Cs gave $\kappa_a = 0.063 - 0.084$. Experiment: [5] $\kappa_a = 0.09(2)$

$$\kappa_a \sim 5\kappa_2$$

[4] V. V. Flambaum, I. B. Khriplovich, O. P. Sushkov Phys. Letts. B **146** 367-369 (1984).

[5] V. V. Flambaum and D. W. Murray, Phys. Rev. C**56**, 1641 (1997); W. C. Haxton and C. E. Wieman, Ann. Rev. Nucl. Part. Sci. **51**, 261 (2001)

C) Hyperfine Interference Contrubution

Interference between the hyperfine interaction H_{hf} and $H^{(1)}$ gives another nuclear spin-dependent correction of the form

$$H^{(\mathsf{hf})} = \frac{G}{\sqrt{2}} \kappa_{\mathsf{hf}} \, \alpha \cdot \mathbf{I} \, \rho(r)$$

$$\begin{aligned} {}^{133}\mathsf{Cs}: \quad & \kappa_{\mathsf{hf}} = 0.0078 \\ {}^{205}\mathsf{Tl}: \quad & \kappa_{\mathsf{hf}} = 0.044 \end{aligned}$$

$$\kappa_{\mathsf{hf}} \sim \tfrac{1}{2} \, \kappa_2$$

Summary of Phenomenology

$$H^{(1)} = \frac{G}{2\sqrt{2}} \gamma_5 \, Q_W \, \rho(r)$$

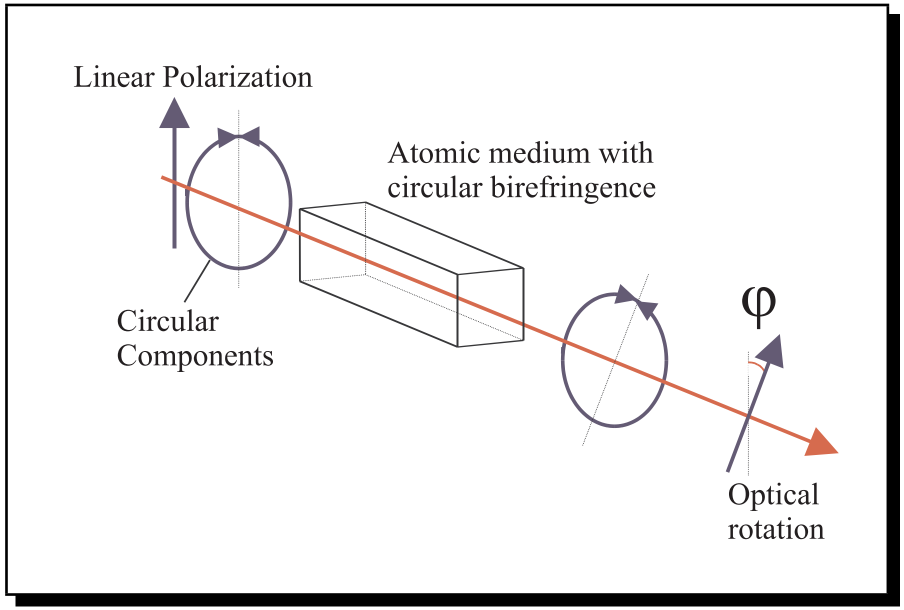
$$H^{(2)} \Rightarrow \frac{G}{\sqrt{2}} \kappa \, \alpha \cdot \mathbf{I} \, \rho(r)$$

where $\kappa = \kappa_2 + \kappa_a + \kappa_{\mathsf{hf}}$.

1. Measure Q_W as a test of Standard Model
2. Measure κ as a test of weak nuclear forces!

Optical Rotation Experiments

Aim is to measure $E_{\mathrm{PNC}} = \langle f|z|i\rangle \propto Q_W$:



The plane of polarization of a linearly polarized laser beam passing through a medium with $n_+ \neq n_-$ is rotated. The rotation angle $\phi \propto R_\phi = \mathrm{Im}\left(E_{\mathrm{PNC}}\right)/M_1$.

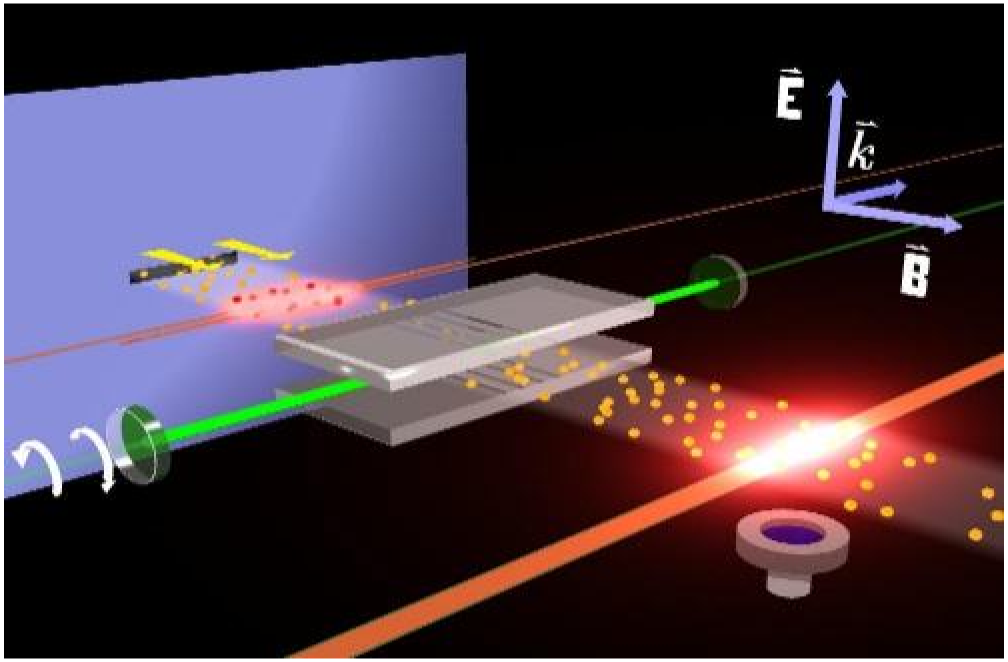
Optical Rotation Experiments

$$R_\phi = \mathrm{Im}\left(E_{\mathrm{PNC}}\right)/M_1$$

Measured values of R_ϕ

Element	Transition	Group	$10^8 \times R_\phi$
^{205}Tl	$^2P_{1/2} - {}^2P_{3/2}$	Oxford (95)	-15.33(45)
^{205}Tl	$^2P_{1/2} - {}^2P_{3/2}$	Seattle (95)	-14.68(20)
^{208}Pb	$^3P_0 - {}^3P_1$	Oxford (94)	-9.80(33)
^{208}Pb	$^3P_0 - {}^3P_1$	Seattle (95)	-9.86(12)
^{209}Bi	$^4S_{3/2} - {}^2D_{3/2}$	Oxford (91)	-10.12(20)

Stark-Interference Experiment



Boulder PNC apparatus: A beam of cesium atoms is optically pumped by diode laser beams, then passes through a region of perpendicular electric and magnetic fields where a green laser excites the transition from the 6S to the 7S state. The excitations are detected by observing the florescence (induced by another laser beam) with a photo-diode.

Stark-Interference Experiments

Evolving values of $R = \text{Im}\left(E_{\text{PNC}}\right)/\beta$ (mV/cm) for ^{133}Cs			
Transition	Group	R_{4-3}	R_{3-4}
$6s_{1/2} - 7s_{1/2}$	Paris (1984)	-1.5(2)	-1.5(2)
$6s_{1/2} - 7s_{1/2}$	Boulder (1988)	-1.64(5)	-1.51(5)
$6s_{1/2} - 7s_{1/2}$	Boulder (1997)	-1.635(8)	-1.558(8)

The vector current contribution from the last row is

$$R_{\text{V}} = -1.593 \pm 0.006$$

$$\text{Im}\left[E_{\text{V}}^{\text{exp}}(6s \rightarrow 7s) \times 10^{11}\right] = -0.8376 \pm (0.0031)_{\text{exp}} \pm (0.0021)_{\text{th}}$$

Other Experiments

Element	Transition	Group
Fr	$7S_{1/2} \longrightarrow 8S_{1/2}$	Stoney Brook
Fr	$7S_{1/2}[F=4] \longrightarrow 7S_{1/2}[F=5]$	Maryland, TRIUMF
Yb	$(6s^2)\,{}^1S_0 \longrightarrow (6s5d)\,{}^3D_1$	Berkeley
Yb	$(6s6p)\,{}^3P_0 \longrightarrow (6s6p)\,{}^3P_1$	Berkeley
Ba^+	$6S_{1/2} \longrightarrow 5D_{3/2}$	Seattle
Dy	$(4f^{10}5d6s)[10] \longrightarrow (4f^9 5d^2 6s)[10]$	Berkeley
Sm	$(4f^6 6s^2)\,{}^7F_J \longrightarrow (4f^6 6s^2)\,{}^5D_{J'}$	Oxford

Calculations of the $6s \rightarrow 7s$ Amplitude in Cs

Units: $i(-Q_W/N) \times 10^{-11} e a_0$

• SD(T) [6]	-0.909 (4)
• CI+MBPT[7]	-0.905
• PTSCI[8]	-0.908 (5)
• PNC-CI [9]	-0.904
• SDCC [10]	-0.907

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[6] S. A. Blundell et al., Phys. Rev. D**45**, 1602 (1992).
[7] M. G. Kozlov, S. G. Porsev, and I. I. Tupitsyn, PRL **86**, 3260 (2001).
[8] V. A. Dzuba, V. V. Flambaum, and J. S. M. Ginges, Phys. Rev. D **66**, 076013 (2002).
[9] V. M. Shabaev et al., Phys. Rev. A **72** (2005)
[10] B. P. Das et al., THEOCHEM **768**, 141 (2006)

Example of a PNC Calculation

$$E_{\text{PNC}} = \sum_n \frac{\langle 7s|D|np\rangle\langle np|H^{(1)}|6s\rangle}{E_{6s}-E_{np}} + \sum_n \frac{\langle 7s|H^{(1)}|np\rangle\langle np|D|6s\rangle}{E_{7s}-E_{np}}$$

"Weak" RPA gives E_{PNC} accurate to about 3%. Therefore, we organize calculation as follows:

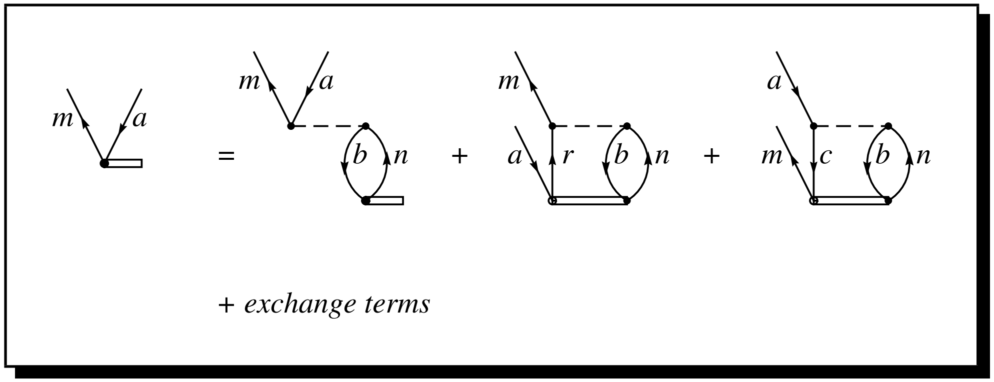
- $n = 6 - 9$ valence states: evaluate matrix elements using SD wave functions (98%)
- $n = 1 - 5$ core states and $n > 10$: evaluate using "weak" RPA amplitudes (2%)

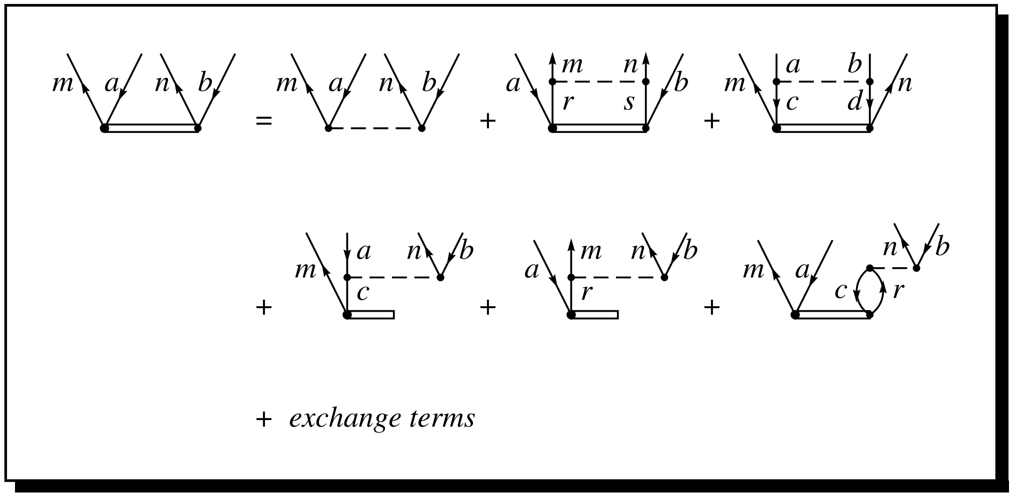
Contributions to PNC Amplitude

Contributions to E_{PNC} in units $-iea_0Q_W/N$.

n	$\langle 7s\|D\|np\rangle$	$\langle np\|H^{(1)}\|6s\rangle$	$E_{6s}-E_{np}$	Contrib.
6	1.7291	-0.0562	-0.05093	1.908
7	4.2003	0.0319	-0.09917	-1.352
8	0.3815	0.0215	-0.11714	-0.070
9	0.1532	0.0162	-0.12592	-0.020
n	$\langle 7s\|H^{(1)}\|np\rangle$	$\langle np\|D\|6s\rangle$	$E_{7s}-E_{np}$	Contrib.
6	-1.8411	0.0272	0.03352	-1.493
7	0.1143	-0.0154	-0.01472	0.120
8	0.0319	-0.0104	-0.03269	0.010
9	0.0171	-0.0078	-0.04147	0.003
$n=6-9$				-0.894(4)
RPA part				-0.015(1)
Total				-0.909(4)

Brueckner-Goldstone Diagrams for the SDCC Equations





Data Analysis

$$E^{\text{exp}}_{\text{PNC}} = E^{\text{th}}_{\text{PNC}} \left[\frac{Q_W}{-N} + \kappa \ \epsilon_{F'F} \right]$$

β (a_0^3)	27.024(80)
$E^{\text{exp}}_{34}/\beta$ (mV/cm)	-1.6349(80)
$E^{\text{exp}}_{43}/\beta$ (mV/cm)	-1.5576(77)
E^{exp}_{34} (10^{-11})	-0.8592(49)
E^{exp}_{43} (10^{-11})	-0.8186(47)
$E^{\text{exp}}_{\text{V}}$ (10^{-11})	-0.8376(37)
$E^{\text{th}}_{\text{PNC}}$ (10^{-11})	-0.9085(45)
Q^{exp}_W	-71.91(46)
κ^{exp}	0.117(16)

Analysis of $6s \rightarrow 7s$ Amplitude in ^{133}Cs

Combining the calculations and the measurements

$$Q_W^{\text{exp}}(^{133}\text{Cs}) = -71.91(46)$$

differs with the standard model value

$$Q_W^{\text{SM}}(^{133}\text{Cs}) = -73.09(3)$$

by 2.5 σ.

Additional Corrections:

- Breit Interaction
- Vacuum Polarization
- αZ Vertex Corrections
- Nuclear Skin Effect

Analysis of $6s \rightarrow 7s$ Amplitude in ^{133}Cs

Combining the calculations and the measurements

$$Q_W^{\text{exp}}(^{133}\text{Cs}) = -71.91(46) \quad \Rightarrow -72.73(46)$$

differs with the standard model value

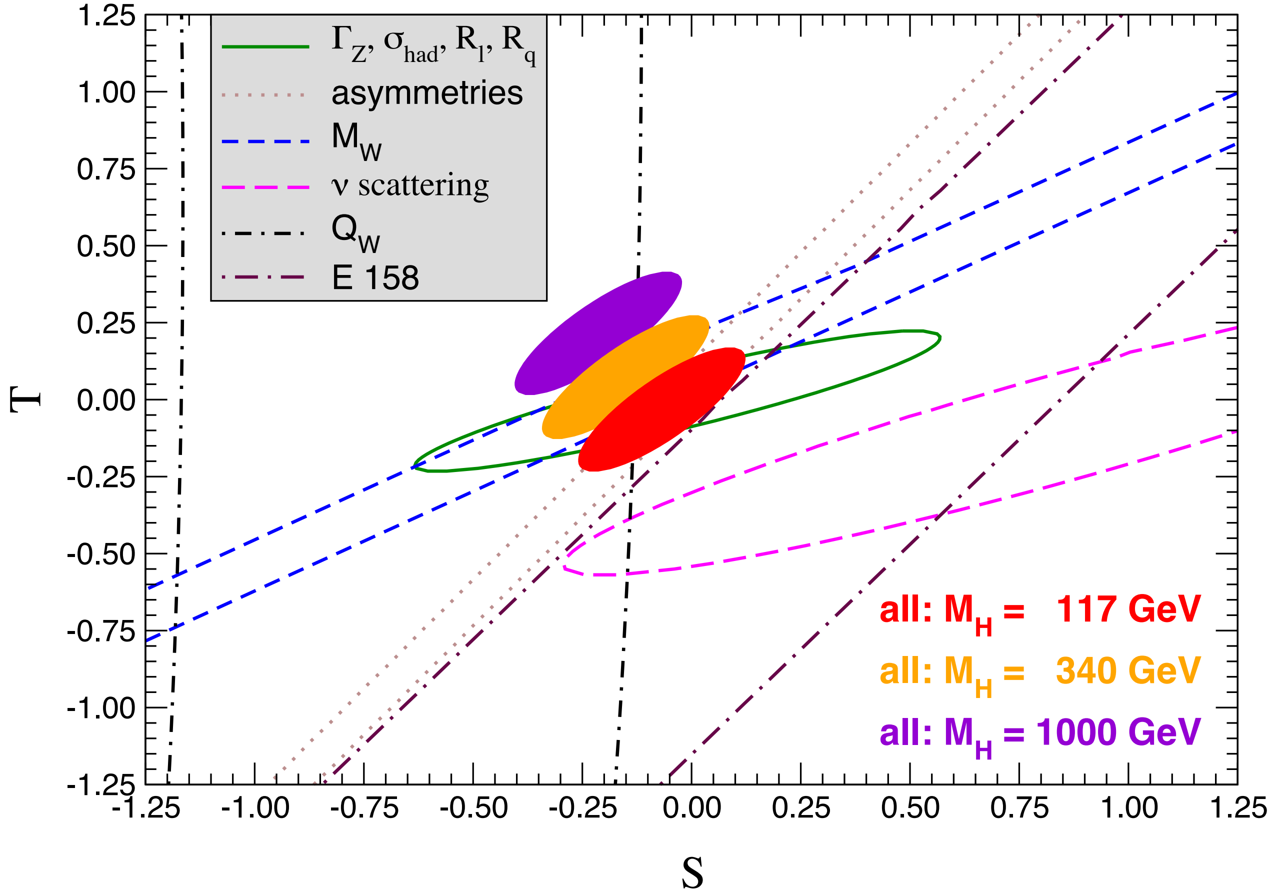
$$Q_W^{\text{SM}}(^{133}\text{Cs}) = -73.09(3)$$

by 2.5 σ. $\Rightarrow$ 0.8 σ.

Additional Corrections:

- Breit Interaction -0.6%
- Vacuum Polarization +0.4%
- αZ Vertex Corrections -0.7%
- Nuclear Skin Effect -0.2%

Constraints on New Physics



Anapole Moment of ^{133}Cs

Group	κ	κ_2	κ_{hf}	κ_a
Safronova and Johnson	0.117(16)	0.0140[1]	0.0049	0.098(16)
Haxton *et al.*	0.112(16)[2]	0.0140	0.0078[3]	0.090(16)
Flambaum and Murray	0.112(16)[4]	0.0111[5]	0.0071[6]	0.092(16)[7]
Bouchiat and Piketty		0.0084	0.0078	

[1]from Haxton *et al.*

[2]from Flambaum and Murray

[3]from Bouchiat and Piketty

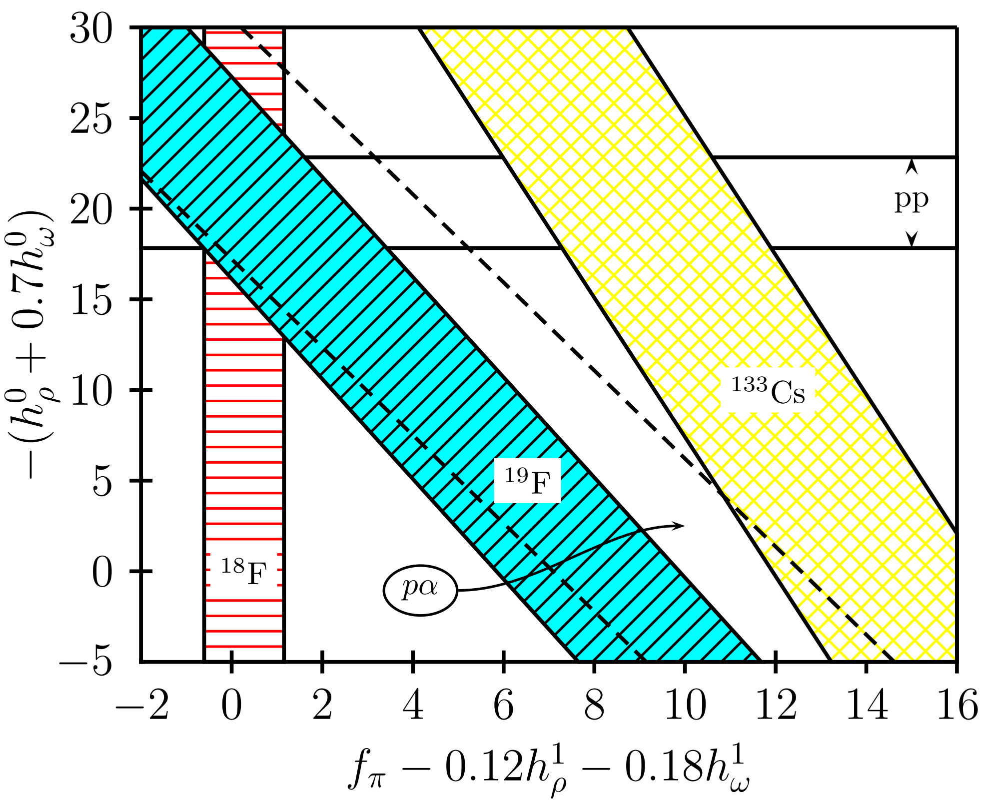
[4]The spin-dependent matrix elements from Kraftmakher are used.

[5]Shell-model value with $\sin^2\theta_W = 0.23$.

[6]This value was obtained by scaling the analytical result from Flambaum and Khriplovich ($\kappa_{\text{hf}} = 0.0049$) by a factor 1.5.

[7]Contains a 1.6% correction for finite nuclear size; the raw value is 0.094(16).

Constraints on Nuclear Weak Coupling Constants[11]



[11] B. Desplanques, J. F. Donoghue, and B. Holstein, Ann. Phys. (NY) **124** 449 (1980); W. C. Haxton and C. E. Wieman, Ann. Rev. Nucl. Part. Sci. **51**, 261 (2001)

Conclusions

- Measurements of the weak charge in heavy atoms provide important tests of the validity of the electroweak standard model and provide limits on possible extensions.

- Measurements of the nuclear anapole moment provide constraints on nucleon-nucleon weak coupling constants that are inconsistent with PNC experiments in light nuclei. **New measurements badly needed!**

- Measurements of PNC in atoms **depend** on precise atomic many-body calculations to provide useful new information concerning weak interaction physics. **Error estimates on calculations of PNC amplitudes are mandatory!**