Book Reviews

Constrained Optimal Control of Linear and Hybrid Systems—Francesco Borelli (Berlin, Germany: Springer-Verlag, 2003). *Reviewed by Hai Lin*

I. Introduction

Nearly every industrial application of automatic control imposes constraints, which may be caused by, for example, actuator saturations, restricted operational regions, or safety considerations. In addition, some control performance requirements can be transformed into certain constraints in the state–space. Unfortunately, systematic methods for handling constrained control problems are relatively sparse, and many industrial applications often resort to *ad-hoc* approaches. Therefore, the last decades have seen increasing research activities in analysis and design of constrained control systems; see, for example, [3] and [5]–[7].

One of the most popular approaches for dealing with control problems with constraints is model predictive control (MPC). MPC falls into the category of sampled data control. However, instead of precomputing a feedback control law or control sequence for the plant as in conventional control, MPC determines the current control action by solving, at each sampling instant, a finite time open-loop optimal control problem, using the current state of the plant as the initial state. The input, output, and state constraints, together with additive disturbances and parametric uncertainties, can be easily handled within the MPC framework by incorporating these constraints into the optimization problem at each sampling instant. This advantage makes MPC very popular in the process industry where satisfaction of constraints is particularly important [7].

Since MPC needs to solve an online optimization problem during each sampling period, it is necessary that the plants being controlled are sufficiently "slow" to permit its implementation. This is the biggest drawback of MPC and limits its applicability to relatively slow and/or small problems. This motivates research efforts with the aim to move all the computations necessary for the implementation of MPC offline. These efforts should largely increase the range of applicability of MPC to industrial problems. The book by Borelli reports the most recent progress achieved by the author and his collaborators along this direction via using multiparametric programming techniques.

Multiparametric programming is a kind of mathematical program dependent on a vector of parameters, which may appear in the cost function and/or in the constraints. The solution of a multiparametric programming is characterized as a function that maps feasible parameters onto optimization solvers. The multiparametric programming is of interest here, since the discrete-time finite time open-loop constrained optimal control problem can be formulated as a multiparametric program, where the sequence of control inputs are the optimization variables and the initial states are the parameters. The advantage of this method is that the multiparametric program can be solved offline and induce a feedback control law for the full range of initial states (instead of a sequence of control signals for a specific initial condition). This provides attractive features for MPC on two accounts. First, it will alleviate the online computation burden, since only function evaluations are necessary at each sampling instant. Second, it also provides an insight into the structure underlying optimization-based controllers. It is interesting to notice that the optimal control law is usually characterized as a piecewise affine state feedback with polyhedral partition of the state–space.

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Digital Object Identifier 10.1109/TAC.2005.851468

II. The Book

The book focuses on two classes of discrete-time dynamical systems, namely constrained linear systems and linear hybrid systems. Hybrid systems are heterogeneous dynamical systems, the behavior of which is determined by interacting continuous-variable and discrete-event dynamics [2]. Linear hybrid systems are referred to as classes of hybrid systems whose continuous-variable dynamics are described by linear differential/difference equations. Hybrid systems have been identified in a wide variety of applications in control of mechanical systems, process control, automotive industry, power systems, aircraft and traffic control, among many other fields. Interested readers may refer to special issues on hybrid systems [2] and [1] for further details.

Several kinds of constrained optimal control problems with cost functions based on $1, 2,$ or ∞ norms for linear systems and linear hybrid systems are investigated in the book. The basic idea is to formulate the constrained optimal control problems into corresponding multiparametric programs, and to characterize the explicit optimal feedback control laws via multiparametric programming. The book is organized into four parts.

The first part of the book contains some preliminary results on multiparametric programming. The author begins with an introduction of basic theory for general nonlinear parametric programming. Then, three efficient geometric algorithms are developed for solving multiparametric linear programs (mp-LP), multiparametric quadratic programs (mp-QP), and multiparametric mixed-integer linear programs (mp-MILP), respectively. The basic idea of the algorithms is to construct a neighborhood region of a given parameter in the parameter space, by using necessary and sufficient conditions for optimality, and then recursively explore the parameter space outside such a region. It is interesting that this kind of region, called critical region, is usually characterized as a polyhedron.

In the second part, the author studies three kinds of constrained optimal control problems for discrete-time linear systems with polyhedral constraints on inputs and states. First, in Chapter 2, the constrained finite time optimal control problems with cost functions based on $1, 2,$ or ∞ norms are formulated as multiparametric linear (1 or ∞ norm based) or multiparametric quadratic (2-norm based) programs. The finite length input sequence is the optimization vector, while the current state of the dynamical systems enters the cost function and the constraints as the parameter vector. It is demonstrated that the solution to all these optimal control problems can be expressed as a piecewise affine state feedback law, accompanied with a polyhedral partition of the state space. Moreover, the optimal control law is continuous, and the value function is convex and continuous. Second, the infinite time constrained optimal control problems with 2-norm based cost function, namely constrained linear quadratic regulator (CLQR) problem, is considered in Chapter 3. The solution for CLQR is given or approximated by a finite time constrained optimal control with sufficiently long horizon.

Since the multiparametric programs can be solved offline, the implementation of MPC is reduced to evaluating the precomputed explicit piecewise affine feedback control law based on current state measurement. Therefore, the characterization of explicit feedback controls for finite time optimal control problems have important consequences for the implementation of MPC. Chapter 4 focuses on MPC problem for constrained linear systems and briefly discusses its stability.

The robust optimal control problem is also considered for constrained linear systems with additive norm-bounded input disturbances and polytopic parametric uncertainties in Chapter 5. It is shown that the robust optimal control law over a finite horizon is a continuous piecewise affine function of the state and the value function is convex and continuous. The piecewise affine feedback control law is based on the polyhedral partition of the state–space, and the number of the polyhedral regions could increase dramatically with the number of constraints in the optimal control problem. Therefore, the author develops two algorithms in Chapter 6 to reduce the storage demands and computational complexity.

The third part (Chapters 7 and 8) of the book is on constrained optimal control for linear hybrid systems. After a short overview of different models for linear hybrid systems, the author uses a piecewise affine (PWA) modeling framework to study the state feedback solution to the finite time optimal control problem for hybrid systems, which can be computed by means of multiparametric mixed integer programming. The optimal control law is shown to be, in general, piecewise affine over nonconvex and disconnected sets. However, this method is basically based on the enumeration of all possible switching sequences, the number of which grows exponentially with the time horizon. Therefore, another hybrid modeling framework, mixed logic dynamical (MLD) system, is employed to avoid such enumeration problems. The key idea of MLD systems is to express the logic relationship in the hybrid systems' description into mixed integer inequalities by transforming Boolen variables into 0–1 integers. It is worth pointing out that the MLD systems are equivalent to other classes of discrete-time hybrid systems such as PWA systems, linear complementarity (LC) systems, and max–min-plus-scaling (MMPS) systems [4]. The MPC problem is also studied for both PWA and MLD systems.

Finally, the applicability of the theoretical results is demonstrated on a mechanical laboratory process and a traction control systems in the application part.

III. Comments

The book is a revised version of the author's Ph.D. dissertation under the guidance of Dr. M. Morari and Dr. A. Bemporad from the Automatic Control Laboratory at ETH-Zurich, Zurich, Switzerland. All the main results presented in the book are contributed originally by the author and his collaborators. The style of the book is clear and concise with many illustrative examples.

Results in the book make interesting contributions to the research areas of constrained optimal control, MPC, and hybrid optimal control. Therefore, the book provides a valuable resource for researchers from both academia and industry. The book is also of interest to the operations research community, since efficient geometric algorithms are developed for multiparametric linear, quadratic, and mixed integer programming. However, some chapters in this book are very short, indicating potential for further development. In this regard, it would be of great interest to include sections discussing future research topics.

Finally, the main goal of the book is the development of efficient geometric algorithms for multiparametric programming and synthesis of explicit feedback control for constrained optimal control problems. Therefore, it would be of great value to develop a software toolbox for calculating the optimization solver.

References

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