Tropical Geometry Syllabus

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Tropical geometry is geometry over the tropical semiring, whose elements are the real numbers together with infinity, but where the addition and multiplication operations are replaced by minimum and addition respectively. Tropical polynomials define tropical hypersurfaces. We'll look in detail at the case of tropical curves in the plane and how they can intersect.

I will introduce tropical varieties as the tropicalizations of varieties defined over fields with valuation. In particular, tropical varieties are the intersection of tropical hypersurfaces. They are also subcomplexes of the Groebner complex of the ideal of the variety, and the valuations of the points of the variety. The tropicalization of an irreducible variety is balanced and connected through codimension 1.

If we take a 1-dimensional tropical variety and remember its edge lengths, we get an abstract tropical curve, i.e. a tropical curve not embedded in any vector space. More precisely, a tropical curve is a graph where the edges have lengths and the leaves have labels, and it is stable if all the vertices other than the leaves have degree at least 3. The set of stable tropical curves of genus g with n labels is itself a topological space. Moreover, when g is 0, this moduli space is a tropical variety, and when g is at least 1, it is still a union of cones.