Practice Exam III Math 228 Fall 05

4. The characteristic polynomial of $\begin{pmatrix} 2 & 0 & 1 \\ 0 & 3 & -2 \\ 1 & 1 & 0 \end{pmatrix}$ is

(a) $-\lambda(2-\lambda)(3-\lambda)$ (b) $-\lambda^3 - 3\lambda^2 + 2\lambda$ (c) $-\lambda^3 + 4\lambda^2 + 2\lambda - 2$

(d) $-\lambda^3 + \lambda^2 - 3\lambda + 8$ (e) $-\lambda^3 + 5\lambda^2 - 7\lambda + 1$

1. Let $W = \text{Span}\{(1,1,1), (2,0,1)\}$ (note that the two vectors are not orthogonal). The distance from $(4,-2,4)$ to W is

(a) 1 (b) 0 (c) 2 (d) $\sqrt{6}$ (e) $\sqrt{3}$

(2)

2. Indicate whether each of the following statements is true or false by placing an x through the letter T or F, respectively. Note that 'true' here means 'definitely true in all cases' and false means 'there are particular examples in which the statement is false'. *Scoring: 3 points for a correct answer, 0 for no answer, -4 for an incorrect answer.*

$\boxed{\text{T}}$ $\boxed{\text{F}}$ If $W \subset \mathbf{R}^n$ is a subspace and $\mathbf{v} \in W$, then $\text{Proj}_W \mathbf{v} = \mathbf{v}$.

~~$\boxed{\text{T}}$ $\boxed{\text{F}}$ A 3×3 matrix is diagonalizable only if it has three distinct eigenvalues.~~

$\boxed{\text{T}}$ $\boxed{\text{F}}$ If A is an $m \times n$ matrix, and $\mathbf{v}, \mathbf{w} \in \mathbf{R}^n$ are vectors such that $\mathbf{v}$ is in the row space of A and $\mathbf{w}$ is in the null space of A, then $\mathbf{v} \cdot \mathbf{w} = 0$.

$\boxed{\text{T}}$ $\boxed{\text{F}}$ An $n \times n$ matrix A is orthogonal if the columns of A form an orthogonal set.

~~$\boxed{\text{T}}$ $\boxed{\text{F}}$ If A is an $n \times n$ matrix, and $\mathbf{v}$ is an eigenvector of A, then $\mathbf{v}$ is also an eigenvector of A^2.~~

~~$\boxed{\text{T}}$ $\boxed{\text{F}}$ If V and W are vector spaces and $T : V \to W$ is a linear transformation, then $T(\mathbf{0}) = \mathbf{0}$.~~

3. Find the least squares solution of the system

$$\begin{pmatrix} 2 & 1 \\ 1 & 0 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 3 \\ 2 \\ 4 \end{pmatrix}.$$

(4)

6. Let $W \subset \mathbf{R}^3$ be the subspace spanned by the vectors $(1, 2, 1)$ and $(-1, 0, 3)$.

- Find a non-zero vector $\mathbf{v} \in W^{\perp}$;
- If $T : \mathbf{R}^3 \to \mathbf{R}^3$ is given by $T(\mathbf{v}) = \mathrm{Proj}_W\, \mathbf{v}$, then T is a linear transformation. Write down the eigenvalues of T and give a basis for each eigenspace. This should not require computation as much as it should thinking.
- Express the standard matrix A for T in the form $A = PDP^{-1}$ where D is a diagonal matrix. In other words, say what P is and what D is.

(5)

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Quiz 10 *November 18, 2004*

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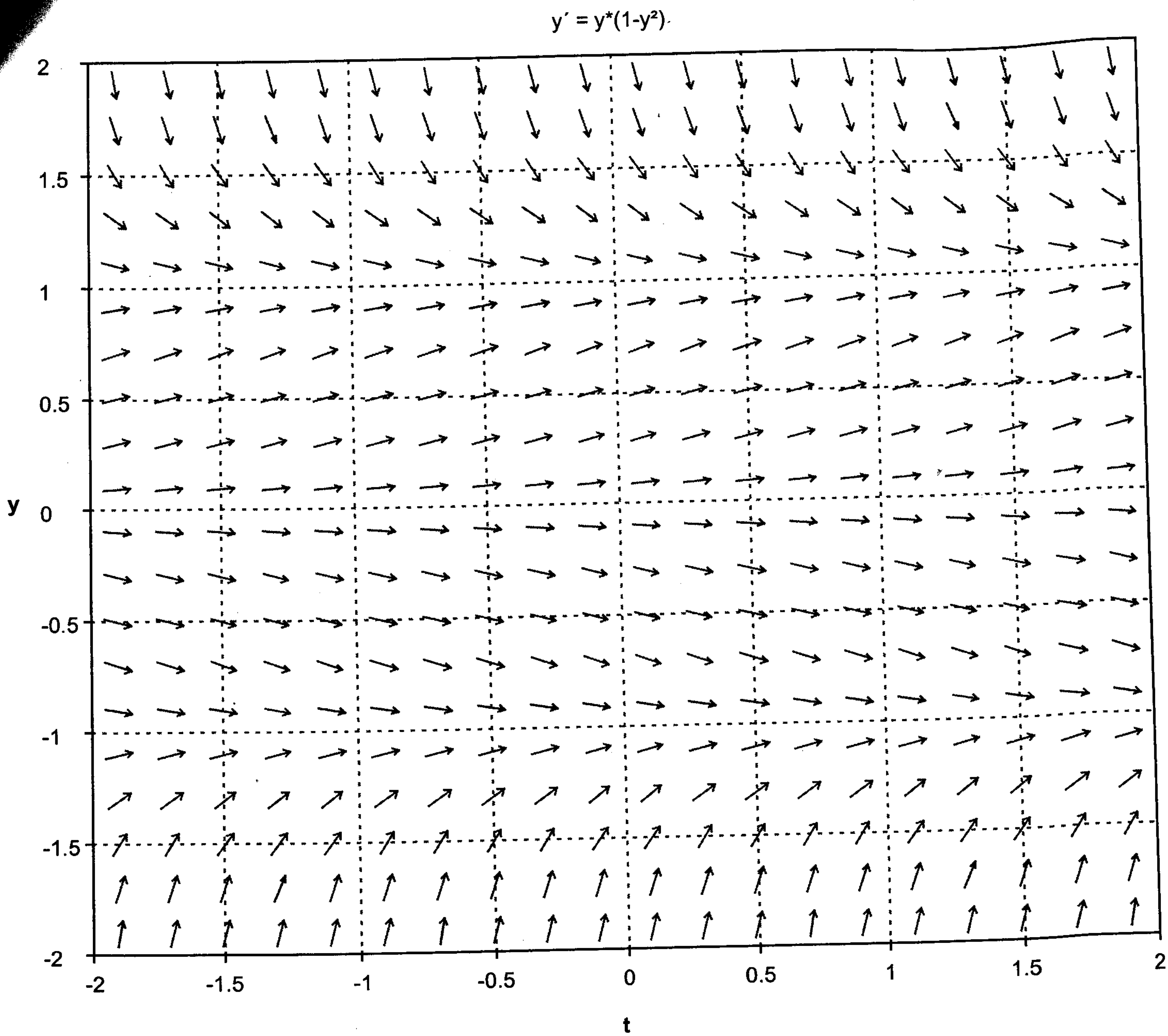
You may use your own calculator. You may not use anything else. You may not pass a calculator to another person.

5.

Show all your work. Erase or cross out any work you do not want graded.

(1) Solve $ty' + y = te^{-t^2}$, $t > 0$.

(2) Determine the critical (equilibrium) points of the equation $y' = y(1 - y^2)$. Classify each critical point as asymptotically stable or unstable. On the attached direction field, sketch several graphs of solutions in the ty-plane.


y′ = y*(1-y²)
y
t
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-2
-1.5
-1
-0.5
0
0.5
1
1.5
2

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(25 points) Solve **any two** of the following three initial value problems. If you attempt all three, clearly indicate which one should not be graded.

6. (a)

$$y' + \frac{y}{t} = \frac{\cos t}{t}, \qquad y(\pi) = 0.$$

(b)

$$(2x - y) + (2y - x)y' = 0, \qquad y(1) = 3.$$

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(c)

$$y'' - 6y' + 9y = 0, \qquad y(0) = 1,\ y'(0) = 0.$$

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points) After an alleged scandal in the math department, there is some gossip on campus. The following differential equation is used to model how the gossip spreads

$$y' = ry(1 - y/K)$$

where r and K are positive constants.

(a) Find any equilibrium solutions and classify them as stable/unstable/semi-stable.

(b) Sketch what the solutions look like.

(c) Explain what the constants r and K mean.

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8 . If the Gram-Schmidt process (the process of making **orthonormal** vectors with the same span) is applied to $\mathbf{v}_1 = (1, 1, 1, 1)$ and $\mathbf{v}_2 = (0, 1, 0, 1)$, the second vector $\mathbf{u}_2$ should be

a) $(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2})$ b) $(-\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, \frac{1}{2})$ c) $(-\frac{1}{2}, \frac{1}{2}, -\frac{1}{2}, \frac{1}{2})$
d) $(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, -\frac{1}{2})$ e) $(\frac{1}{2}, \frac{1}{2}, -\frac{1}{2}, -\frac{1}{2})$.

9 ~~12~~. The general solution of the equation $xy' - y = x^2$ is

a). $\frac{x^3}{3} + C$

b). $\frac{x^3}{2} + Cx$

c). $x^2 + Cx$

d). $x^2 + \frac{C}{x}$

e). $\frac{x^3}{3} + \frac{C}{x}$.

10 ~~13~~. Suppose a population N satisfies the logistic equation

$$N' = -N(1-N)(3-N) \text{ (millions/year)}.$$

If the initial population is $N(0) = 2$(million) , what is the limiting value in millions of the population over a long period of time?

a) 2

b) 1

c) 3

d) 0

e) ∞.

11 ~~14~~. A young person with no initial capital invests 1000 dollars per year at an annual rate of 5% return. Assume that the investments are made continuously and the return is compounded continuously. What is the equation for the total sum $S(t)$ accumulated at time t?

a) $S'(t) = (0.05)S(t)$.

b) $S'(t) = (0.05)S(t) - 1000$.

c) $S'(t) = -(0.05)S(t) + 1000$.

d) $S'(t) = (0.05)S(t) + 1000t$.

e) $S'(t) = (0.05)S(t) + 1000$.

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12 Let y be the solution satisfying

$$\frac{dy}{dx} = -\frac{2x+3y}{3x+4y}, \qquad y(0) = 1$$

then y satisfies

a). $2y^2 - 3xy + x^2 = 2$

b). $2y^2 + 3xy + x^2 = 2$

c). $2y^2 + 3xy - x^2 = 2$

d). $2y^2 - 3xy - x^2 = 2$

e). $2y^2 + 6xy + x^2 = 2$

13 Compute the Wronskian of the functions

$$y_1 = \cos 2x, \qquad y_2 = \sin 2x$$

a). 1

b). 2

c). 0

d). -2

e). -1

(13)

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(14) 15. Which of the following pair of functions is linearly dependent:

a). $f(x) = 1 + 2x, \quad g(x) = 1$

b). $f(x) = e^x, \quad g(x) = e^{x+1}$

c). $f(x) = \sin x, \qquad g(x) = \sin 2x$

d). $f(x) = e^x, \qquad g(x) = e^{-x}$

e). $f(x) = 3x - 5, \qquad g(x) = 9x - 10$

18. Find the solution for the given initial value problem

$$y'' - 2y' - 3y = 3e^{2t}$$
$$y(0) = 1$$
$$y'(0) = 0.$$

(a) $y = e^{3t} - e^{-t} + e^{2t}$

(b) $y = e^{3t} + e^{-t} + 2e^{2t}$

(c) $y = 2e^{3t} + 2e^{-t} - e^{2t}$

(d) $y = e^{3t} + e^{-t} - e^{2t}$

(e) $y = e^{3t} - 3e^{-t} + e^{2t}$

15 [illegible]. The general solution to the equation $u'' + 4u' + 5u = 0$ is

a). $u = c_1 e^{-2t} + c_2 e^{-3t}$

b). $u = c_1 \cos 2t + c_2 \sin 2t$

c). e). $u = c_1 e^{-t} \cos 2t + c_2 e^{-t} \sin 2t$

d). e). $u = c_1 e^{2t} \cos t + c_2 e^{2t} \sin t$

e). $u = c_1 e^{-2t} \cos t + c_2 e^{-2t} \sin t$

16 [illegible]. Give an interval where the initial value problem is certain to have a unique solution.

$$ty' + \frac{t+1}{t+2} y = \frac{1}{3+t}$$
$$y(1) = -1$$

(Do not attempt to solve it.)

a) $(-2, \infty)$

b) $(-3, \infty)$

c) $(0, 2)$

d) $(-3, -2)$

e) $(0, \infty)$.

15

17) ~~23.~~ The solution u of the differential equation

$$\frac{du}{dt} = u(1-u), \qquad u(0) = \frac{1}{2}$$

is

$$a).\frac{e^t}{1+e^t}$$

$$b).\frac{e^{-t}}{1+e^{-t}}$$

$$c).\frac{e^{-t}}{1+e^t}$$

$$d).\frac{-e^{-t}}{1-2e^{-t}}$$

$$e).\frac{1+e^t}{e^t}$$

18) ~~22.~~ The general solution of the differential equation

$$\frac{dy}{dx} = \frac{1-2x}{y}$$

is:

(a) $y^2 - 2x + x^2 = C$

(b) $y^2 - x + x^2 = C$

(c) $y^2 = 2x - 2x^2$

(d) $y^2 - 2x + 2x^2 = C$

(e) $y = x - x^2 + C$