Answer Key 1

MATH 20–580: Linear Alg. and Diff. Eq. Name:________________________

Exam IIi *October 25, 2005* Instructor:________________________

Record your answers to the multiple choice problems by placing an × through one letter for each problem on this page. There are 6 multiple choice questions worth 6 points each and 4 partial credits problems worth 12 points each. You start with 16 points. On the partial credit problems try to simplify your answer and indicate your final answer clearly. *You must show your work and all important steps to receive credit.*

You may use a calculator if you wish.

HONOR CODE PLEDGE: As a member of the Notre Dame community, I will not participate in or tolerate dishonesty.

PLEASE SIGN:________________________

1.	a	•	c	d	e	4.	a	b	c	•	e
2.	a	b	c	•	e	5.	a	•	c	d	e
3.	a	•	c	d	e	6.	•	b	c	d	e

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1. Suppose V is the set of all vectors in $\mathbb{R}$ of the form

$$\begin{bmatrix} 3b+2c \\ b+c \\ 2b+3c \end{bmatrix}.$$

Then

(a) $V = \text{Span}\{ \begin{bmatrix} 5 \\ 2 \\ 5 \end{bmatrix}, \begin{bmatrix} 10 \\ 4 \\ 10 \end{bmatrix} \}$

(b) $V = \text{Span}\{ \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix} \}$

(c) $\dim V = 3$

(d) $\dim V = 1$

(e) $V = \text{Span}\{ \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}, \begin{bmatrix} 7 \\ 2 \\ 8 \end{bmatrix} \}$

2. Suppose A is a 5×7 matrix and dim Nul A $= 3$. Then the dimension of the row space of A is

(a) 7

(b) 3

(c) 5

(d) 4

(e) 1

3. Let $A = \begin{bmatrix} 1 & 7 & 2 \\ 0 & 0 & 0 \\ 1 & -17 & 3 \end{bmatrix}$ Then the characteristic equation of A is

(a) $\lambda^2 - 4\lambda - 1$

(b) $-\lambda^3 + 4\lambda^2 - \lambda$

(c) $\lambda^3 + 2\lambda^2 - 2\lambda - 1$

(d) 0

(e) $\lambda(1-\lambda)(3-\lambda)$

4. Let A be a 4×4 matrix with characteristic polynomial $(\lambda - 1)(\lambda + 3)(\lambda + 2)(\lambda - 3)$. Find $det(A)$. Hint: Think about the definition of the characteristic polynomial.

(a) -18

(b) 0

(c) -9

(d) 18

(e) 9

5. Let $A = \begin{bmatrix} 3 & a & -4 & c \\ 0 & 2 & 1 & -7 \\ 0 & 0 & 3 & b \\ 0 & 0 & 0 & 2 \end{bmatrix}$. Which of the following is *true*

(a) A is diagonalizible iff $a = 7$, $b = -1$, and $c = 2$.

(b) A is diagonalizible iff $a = 4$ and $b = -7$.

(c) A is diagonalizible iff $a = 3$ and $b = 3$.

(d) A is diagonalizible for all values of a, b, c.

(e) A is never diagonalizible.

6. The equation $x_1 + x_2 + x_3 = 0$ defines a plane V in $\mathbf{R}^3$. In this plane consider the two bases $\mathcal{B} = \{ \begin{bmatrix} 1 \\ 2 \\ -3 \end{bmatrix}, \begin{bmatrix} 4 \\ -1 \\ -3 \end{bmatrix} \}$ and $\mathcal{C} = \{ \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} \}$. Then the change-of-coordinates matrix from $\mathcal{B}$ to $\mathcal{C}$, $P_{\mathcal{C} \leftarrow \mathcal{B}}$, is

(a) $\begin{bmatrix} 2 & -1 \\ 1 & 4 \end{bmatrix}$.

(b) $\begin{bmatrix} 0 & 1 \\ 1 & 4 \end{bmatrix}$.

(c) $\begin{bmatrix} 2 & 1 \\ 1 & 4 \end{bmatrix}$.

(d) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$.

(e) $\begin{bmatrix} 1 & 1 \\ 2 & 4 \end{bmatrix}$.

7. Let $\mathbf{v}_1 = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$, $\mathbf{v}_2 = \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix}$ $\mathbf{v}_3 = \begin{bmatrix} 1 \\ x \\ 1 \end{bmatrix}$. Find x so that $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ is not a basis for $\mathbf{R}^3$.

8. Let $V = M_{2,2}$ be the vector space of 2×2 matrices. Let $\mathbf{v}_1 = \begin{bmatrix} 1 & 2 \\ 5 & 2 \end{bmatrix}$, $\mathbf{v}_2 = \begin{bmatrix} 4 & 3 \\ 1 & 6 \end{bmatrix}$, $\mathbf{v}_3 = \begin{bmatrix} -2 & 1 \\ 9 & -2 \end{bmatrix}$. Find a subset of $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ which is a basis for $\text{Span}\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$. You must justify your answer.

9. Let A be the matrix

$$\begin{bmatrix} 1 & -1 & 1 \\ 0 & 2 & -4 \\ 0 & 0 & 4 \end{bmatrix}.$$

If possible, find matrices P and D so that $P^{-1}AP = D$ where D is diagonal.

10. Let T be the linear transformation from upper triangle 2×2 matrices to the lower 2×2 matrices that maps $\begin{bmatrix} a & b \\ 0 & c \end{bmatrix}$ to $\begin{bmatrix} 2a & 0 \\ -b & 3c \end{bmatrix}$. Find the basis for T relative to the bases $\{\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}\}$ and $\{\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}\}$.